

**Direct current simulation in acrylic box using 2D finite different methods**

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**Direct Current Simulation in Acrylic Box**

**Using 2D Finite Different Methods**

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**Abstract.** In order to investigate or study the structure of the subsurface using direct current it is better if the model validated using real small model using low current with low voltage, especially if the domain investigated doesn’t have data to validate. With this real model it is easy to change the structure of the materials and acquire data, and also low cost. We design the box using acrylic because it is easy to build, lighter, and cheaper. The dimension of the box is 100 cm x 50 cm x 50 cm (x, y, z). Because this box is different from the real subsurface that doesn’t have any bound so the simulation must be built to understand the properties of the box. In this paper we were modeling the subsurface homogenous and layered heterogeneous material. The simulation is accomplished by using finite different methods order one in two dimensions (2D) with boundary using acrylic resistivity. Firstly, the simulation is done using homogenous material and the result is agreed with analytical methods. After that we compute the two parallel layers with different materials. The result shows that the potential distribution between electrodes is distribute to all surface at top, bottom, left, and right.

**INTRODUCTION**

Resistivity method is successfully used to investigate variety scale medium from cell scale [1] to earth scale [2]. The variety of the applications is wide, from environmental and engineering, hydrological, archeological and mineral exploration surveys [3]. In order to investigate the subsurface, the result from computer analysis must be validated using the data from other geophysical method, or from other technical or engineering data. In order to research contaminated site at Lernacken, Sweden, Dahlin *et al.* [4] validate the result using geological data. Chambers *et al.* [5] is using geographical data and resistivity mapping from the past to validate environmental contamination by waste-disposal site. Calendine *et al.* [6] built an automation system to detect leak from buried tanks at the Hanford nuclear site, and in order to validate the result, the ground is injected by using water.

The problem validates in resistivity survey is if the domain that investigated doesn’t have data to validate. It happens if the field is new site and there has not someone investigate it yet in the past. To solve this problem there are two ways to do, first take the sample from the ground, the second create a small scale real model. Even though the first is the best way to validate, it is expensive because we must dig the ground by using heavy duty machine, especially if the sample at the deep down, beside that sometime the field is barrier to dig. If we have trouble by using the first way, we must use the second way. It is extreme inexpensive compared to the first way. Dahlin and Zhou [7] show that all resistivity methods surveys are subject to noise and made the result inversion that have a little different result between them. It means the result from the analysis is not accurate so the small real model based in it is not accurate too. But it is much better if we have the data to validate, even though the data is not accurate, than we don’t have any data at all.

According to Ohm laws the resistivity of medium is independent from voltage and current, in theory we could create the same small real model based on the larger scale real model. It will give us a big benefit when we must investigate a model that produced by computer and we want to validate it using a small real model. By using this model we could easily rearrange medium and acquired data.

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In order to realize the real small model we will build the model using acrylic box. The reasons we choose acrylic box because it is cheap and has high value resistivity. Beside that acrylic is easy to build as a box, and if it is broken it is not as dangerous as glass. In this paper we do preliminary work forward modeling the box using finite difference in 2D. We choose the finite different model because it is easy to build and high speed, and shows an accurate similar result to another accurate method like the finite element [8]. In this paper we just need to know the potential distribution of the box without worrying about the accurate result. This potential distribution in the acrylic box is important to know because the space of this small scale model is bounded in all direction, in contrast to the survey field which is done at unbounded space. The bounded domain could need a new method to analyze the structure in the box.

**FUNDAMENTAL RESISTIVITY EQUATION**

The continuity equation of flow steady current in a non uniform medium given by:

−∇ ⋅ **J** = ∂*Q* ,

∂*t*

**J** is current density and *Q* is volume density of charge. Using Ohm laws,

**J** = σ **E**,

(1) (2)

where σ is conductivity. If **E** = ∇φ

and insert it in Equation 2, it can be write as:

−∇ ⋅[σ∇φ ] = ∂*Q* ,

∂*t*

(3)

where *ϕ* is scalar potential. Expanding Equation 3 in Cartesian axes,

∂  ∂

 ∂ 

∂  ∂  ∂ 

φ ( *x*, *y*, *z*)

σ ( *x*, *y*, *z*)  +

φ ( *x*, *y*, *z*)

σ ( *x*, *y*, *z*)  +

φ ( *x*, *y*, *z*)

σ ( *x*, *y*, *z*) 

∂*x* 

∂*x* 

∂*y* 

∂*y* 

∂*z* 

∂*z* 

(4)

+*q*δ ( *xs* , *ys* , *zs* ) = 0

where *q* = ∂*Q* / ∂*t*

is the current density, δ is delta dirac, and *s* indices is the location of point source current. In two

dimension using the assumption there is no variation in conductivity in the *y* direction, where,

∂ σ ( *x*, *y*, *z*) = 0 , (5)

∂*y*

re-writing Equation 4 and according to Equation 5, it could have:

∂  ∂φ ( *x*, *z*)  ∂  ∂φ ( *x*, *z*) 

σ ( *x*, *z*)  + σ ( *x*, *z*)  + *q*δ ( *x* , *z* ) = 0 . (6)

∂*x* 

∂*x* 

∂*z* 

∂*z*  *s s*

Equation 6 is for flow steady current using one point current source, commonly two current sources are used which one act as a source and other as a sink. The equation of the two point current sources could be written as follows:

∂  ∂φ ( *x*, *z*)  ∂  ∂φ ( *x*, *z*) 

σ (*x*, *z*)  + σ ( *x*, *z*)  + *q*[δ ( *x* , *z* ) − δ ( *x* , *z* )] = 0 , (7)

∂*x* 

∂*x* 

∂*z* 

∂*z* 

*i i o o*

(*xi*, *zi*) and (*xo*, *zo*) are the position of the current and sink source, respectively.

**RESISITIVITY 2D FINITE DIFFERENCE DISCRETIZED**

In order to compute direct current using 2D finite difference problem we follow Mufti’s [9]. We discretized differential term in Equation 7 using Taylor expansion using central difference as follows:

∂*f* ( *x*, *z*) =  *f* ( *x* + *h*, *z*) − *f* ( *x* − *h*, *z*) + *E* (*h*)

∂*x h*

(8)

*h* is the mesh node distance, *E*(*h*) is the error term. The *y*-axis notation is removed. Applying Equation 8 to *x* term,

 ∂φ ( *x*, *z* ) 

∂ σ ( *x*, *z*) 

σ ( *x* + *h*, *z*) ∂φ ( *x* + *h*, *z* ) − σ ( *x* − *h*, *z*) ∂φ ( *x* − *h*, *z* )

  ∂*x*  = ∂*x* ∂*x*

∂*x h*

Applying Equation 8 into ∂φ(*x* + *h*, *z*) / ∂*x*

(9)

∂φ ( *x* + *h*, *z*) = φ ( *x* + 2*h*, *z* ) − φ ( *x* − 2*h*, *z* )

∂*x h*

(10)

For simplicity the indices of location is changes to *i*, *k* for *x*, *z* respectively and differential notation change to prime symbol, Equation 10 then will be:

φ − φ

φ′ = *i* + 2, *k i* − 2, *k*

*i* +1,*k h*

(11)

If the Equation 11 compared to the stencil in Figure 1, the location of *x*+2*h* is *i*+2. It could be a great benefit if the node distance for differential chooses half of *h*. Inserting *h* =*h*/2 into Equation 10 gives:

φ − φ

φ′ = *i* +1, *k i* −1, *k*

*i* +1/ 2,*k h* / 2

Inserting Equation 12 into Equation 7 and solve for *z* axes, it give the result as,

(12)

where,

α*ip*φ*i* +1, *k* + α*im*φ*i* −1, *k* + α*kp*φ*i* , *k* +1 + α*km*φ*i* , *k* −1 − α*t* φ*i* , *k* + *qi* , *k* = 0 , (13)

α = σ

*h*−2 = (σ + σ

) / 2*h*2

*ip i* +1/ 2, *k i* , *k i* +1/ 2, *k*

α = σ

*h*−2 = (σ + σ

) / 2*h*2

*im i* −1/ 2, *k i* , *k i* −1/ 2, *k*

α*kp* = σ *i* , *k* +1/ 2

α = σ

*h*−2 = (σ

*h*−2 = (σ

*i* , *k* + σ*i* , *k* +1/ 2

+ σ

) / 2*h*2

) / 2*h*2

(14)

*km i* , *k* −1/ 2

*i* , *k i* , *k* −1/ 2

α*t* = α*ip* + α*im* + α*kp* + α*km*

Equation 13 and Equation 14 are the base equation for computing 2D finite difference inside the box. The next step is to apply the equations into the boundary condition.

**The Domain Problem**

The domain of the problem is an acrylic box with the dimension of 100 cm x 50 cm x 50 cm (*x*, *y*, *z*). In two

dimensions we use 100 cm and 50 cm for *x*, *z* respectively. The acrylic resistivity for 2D problem is 1.9 × 1015 ohm- cm, thus the surface resistivity. The medium in a box is sandstone with resistivity value is 5 × 105 ohm-cm. The

location of electrode is at (0, 45) and (0, 55) denote as *A* and *B*, respectively. *A* is input current and *B* is output

current. The arrangement of the box is shown in Fig. 2.

According to Fig. 2 the boundary condition of all edge is Neuman boundary condition type, where:

∂φ = 0

∂*r r* = *at edge*

(15)

At the surface where the elements are along (2, *k*), which 2≤*k*≤*K*-2, and *K* is the maximum value of *k*. In this

discussion we exclude the top-left, right-top, left-bottom, and right-bottom corner. Let’s introduce the fictitious row

elements above the surface of the ground, as shown in Fig. 3, such that:

It makes,

At elements along (1, *k*) it must have,

σ *i* −1, *k* = σ *i* +1, *k*

α*im* = α*ip* .

(16) (17)

This condition is satisfied by,

∂φ

∂*z z* = 0

= 0.

(18)

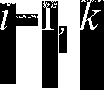
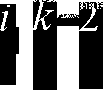
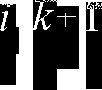
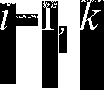
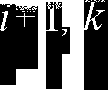
φ*i* − *h*, *k* = φ*i* + *h*, *k* .

Inserting Equation 16, Equation 17 and Equation 19 into Equation 13, then Equation 13 can be replaced by:

α*kp*φ*i* , *k* +1 + α*km*φ*i* , *k* −1 + 2α*ip*φ*i* +1, *k* − α*t* φ*i* , *k* + *qi* , *k* = 0.

(19) (20)

(0, 0)



100cm

A B

10cm

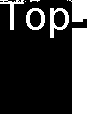
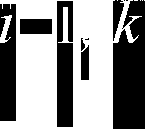
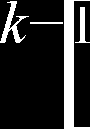
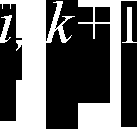
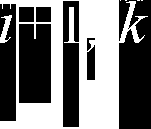
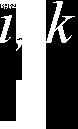
(0, K)

Sandstone

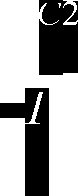
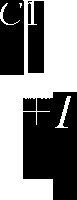
50cm

Acrylic

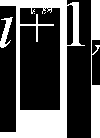
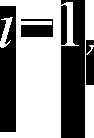
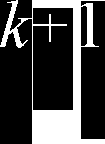
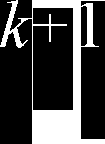
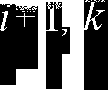
(I, 0)



**FIGURE 1**. Stencil at position *i*, *k* **FIGURE 2**. The acrylic box in two dimensions



(I, K)



**FIGURE 3**. Stencil with fictitious node at surface



**FIGURE 4**. Stencil at the top-left corner

**FIGURE 5**. An arbitrary geometric factor of two electrodes

Since this area is the location of current injection, the *q* value is appearing at this location. The current injected commonly as amperes (*Ic*) whereas the current density expressed as amperes per unit area. According to the rises of Equation 12 where the distance of difference is *h*/2, the area of current flow at (*i*, *k*) is *h*2 and it gave,

*qi* , *k*

=  *Ic* .

*h*2

(21)

Equation 21 is the current density inside the medium, and if the current injected at the surface, the area is halves and the current density will be:

*q*0, *k*

= 2*I c* .

*h*2

(22)

Using the same way, the equation at along left edge (*i*, 1), where 1≤*i*≤ *I*-1 it gives,

2α *kp*φ*i* , *k* +1 + α*im*φ*i* −1, *k* + α *km*φ*i* , *k* −1 − α*t*φ*i* , *k* + *qi* , *k* = 0.

At along right edge (*i*, *K*-1), where 1≤*i*≤ *I*-1 it gives,

α*im*φ*i* −1, *k* + 2α *km*φ*i* , *k* −1 + α*ip*φ*i* +1, *k* − α*t* φ*i* , *k* + *qi* , *k* = 0.

At along bottom edge (*I*-1, *k*), where 1≤*k*≤*K*-1 it gives,

α*kp*φ*i* , *k* +1 + 2α*im*φ*i* −1, *k* + α *km*φ*i* , *k* −1 − α*t*φ*i* , *k* + *qi* , *k* = 0.

(23) (24) (25)

At the top-left corner as seen in Fig. 4, lets introduce two fictitious node (*i*-1, *k*) and (*i*, *k*-1), in this place it gives:

σ *i* −1, *k* = σ *i* +1, *k*

It makes,

As top surface, the boundary condition is,

σ *i* , *k* −1 = σ *i* , *k* +1

α*im* = α*ip*

α*km* = α*kp*

∂φ = ∂φ

= 0.

(26)

(27) (28)

∂*x x* = 0

∂*z z* = 0

Using Equation 26, Equation 27 and Equation 28, the equation at top-left corner becomes,

2α*ip*φ*i* +1, *k* + 2α*kp*φ*i* , *k* +1 − α*t* φ*i* , *k* + *qi* , *k* = 0 , (29)

Using the same way, at top right corner we got,

2α*ip*φ*i* +1, *k* + 2α*kp*φ*i* , *k* −1 − α*t* φ*i* , *k* + *qi* , *k* = 0 , (30)

At left-bottom corner, At right-bottom corner,

2α*im*φ*i* −1, *k* + 2α*kp*φ*i* , *k* +1 − α*t*φ*i* , *k* + *qi* , *k* = 0 , (31)

2α*im*φ*i* −1, *k* + 2α*km*φ*i* , *k* −1 − α*t*φ*i* , *k* + *qi* , *k* = 0 , (32)

**Validating the Computation**

The program to compute the potential distribution in this paper is written in Matlab. In order to validate the program, firstly it will be tested by using analytical computation in homogeneous medium using the relation,

ρ = *G*  ∆*V*

*a I*

(33)

where ρ*a* is apparent resistivity, *G* is geometric factor, ∆*V* is potential, and *I* is current. If there are two electrodes we

could measure resistivity using arbitrary configuration using,

*G* = 2π 1

( *r*1 − *r*2 − *r*3 + *r*4 )

1 1 1 1

(34)

The configuration of Equation 34 could be seen in Fig. 5.

If the medium is homogeneous half-space, ρ*a* will be the true intrinsic resistivity of the medium. If the lower half

of the medium has inhomogeneous conductivity, ρ*a* become apparent resistivity. If *P*1 in Fig. 5 placed at midpoint in

the line joining *C*1 and *C*2 its make *r*1 = *r*3 and Equation 34 become:

*G* = 2π 1 ,

( 1 − 1 )

(35)

*r*4 *r*2

and the potential at *P*2 relative to midpoint of electrodes could be find using

φ*P* 2

= ρ *I*  1 − 1  .

(36)

2π  *r*4

 

*r*2 

In this test the program is validated at homogeneous medium with the dimension of 1×1 m, *C*1 and *C*2 at 0.4 m and 0.6 m respectively, *ρ* = 100 ohm-m, current *I* = 25 mA/unit length. In this test we use Equation 13 for the inside of the medium and Equation 20 for the surface, and *h* = 0.05. As a result, *r*1 = *r*3 = 0.5 m and *r*2 and *r*4 vary depend on *P*2 position. *P*2 position is advanced with step *h* from 0 to 1 m. Figure 6 shows the arrangement of this test.

The result of computational is shown in Fig. 7. The result is in good agreement with analytical result even

though not so great agreement especially at near electrode. It happens because the *h* is coarse. This coarse makes the computation at near electrode not accurate because analytically the potential is increases rapidly with the small distance. It means there will be some potential value that is not covered by the *h*.

6

Finite Difference

Analytic

4

2

0



Potential, V

-2



-4

-6

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Horizontal distances, m

**FIGURE 6**. Arrangement of the 1st test **FIGURE 7**. The result of computation compare to analytic result

**THE APPLICATIONS AND DISCUSSION**

After successfully validate using the analytical result in homogeneous medium, we tested this result into acrylic box. In this test we try do the simulation in homogeneous medium and two parallel layers.

**Homogeneous Medium**

Figure 8 shows the potential structure in unbounded surface and box. In unbounded surface we create very large domain to simulate infinitive surface and capture the result at 100 cm x 60 cm. From Fig. 8(a) the line potential is following the curve and will go out at the top surface. It is expected because the governed equation is 1/*r* as seen in Equation 36. In Fig. 8(b) the potential curves that cannot reach the top surface will point toward to the left, right and bottom edge. These states are also expected because the equations used at those locations are from Equation 20, and Equation 23 through Equation 29. Those equations are governing the potential at the surface. Its looks like the path point toward the nearest surface.

**Two Parallel Layers**

After testing in homogeneous medium, the program applied to two parallel layers. Using domain problem as

shown in Fig. 2, the resistivity top layer is 1.9 × 1013 ohm-m with the thickness of 30 cm. The second layer has resistivity of 1.0 × 104 ohm-m, which is the value of igneous and metamorphic rocks. The result is shown in Fig. 9

which similar to result in Fig. 8(b). The potential curves that cannot reach the top surface will point toward another

surface at left, right, and bottom surface. The potential distribution at the top surface between electrodes looks like copying at the bottom surface at larger scale position. The different result is seen at left and right potential distribution, at the same location it looks like the potential isn’t mirror.

In the second simulation the electrodes is located at the left boundary at 20 cm and 40 cm. The result is shown in Fig. 10. The result shows as in the first simulation where the potential distribution at the top surface copying at the bottom surface at larger scale position. The potential distribution between electrodes now concentrated at the left

surface.

0

0.1

-0.01295

0.2

0.3

Depth, m

0.4

0

0.1

0.1289

0.2

0.3

0.93153

Depth, m

0.4

0.5

0.5

0.6

0.6

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Horizontal distance, m

0 0.2 0.4 0.6 0.8 1 1.2

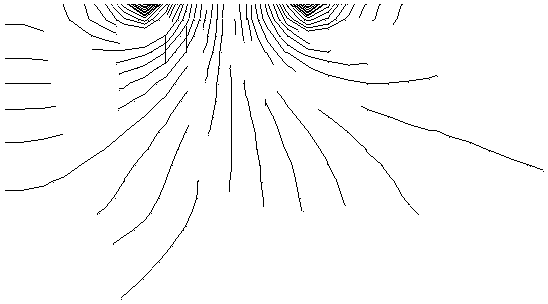
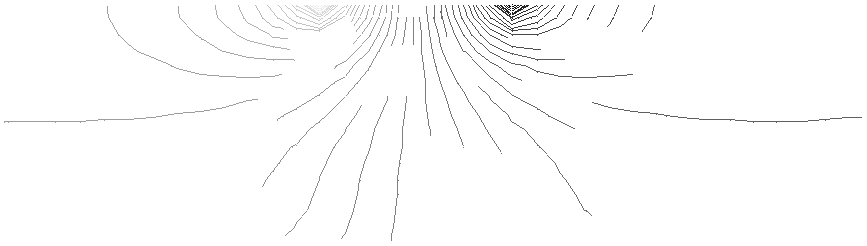
-0.70091

Horizontal distance, m

(a) (b)

**FIGURE 8**. Potential distribution in (a) unbounded surface, (b) box

0 0



0.38119

0.1

0.1

0.2

0.2

0.3

0.3

0.14176

0.4

0.4

0.5

0.5

0.6

0.6

0.7

0 0.2 0.4 0.6 0.8 1 1.2

0.7

0 0.2 0.4 0.6 0.8 1 1.2

**FIGURE 9.** Two parallel layers potential distribution in box

**FIGURE 10**. Two parallel layers potential distribution in a box with the electrodes at near one sided boundary

0

0.1

0.2

-2.77606

-2.36734

0.3

-3.18479

-2.77606

0.4

0.5

0.6

0.7

0 0.2 0.4 0.6 0.8 1 1.2

**FIGURE 11**. Two parallel layers potential distribution in a box with the electrodes at near boundary

In the third simulation the electrodes distance are expanded at the left and right boundary at 20 cm and 100 cm. The result is shown in Fig. 11. The result is again shows that the potential distribution at the top surface is almost the same as at the bottom surface and some portion potential distribution are point toward the left and right surface.

**CONCLUSIONS**

-3.18479

The potential distribution in an acrylic box from two electrodes at the top surface using one or more layer shows that the potential distribution will be spread through all of surface at the left, right, and bottom surface. The distribution at the bottom layer is taking most part of the top surface distribution. Those results are interesting in order to know the structure inside a box, and it can be investigated using potential distribution at all surfaces.

**THE FUTURE WORK**

In the future we will use 3D using finite difference methods or using another method like finite element to better understand the potential distribution of the box. In this paper we neglected the medium of the bottom box. We will change other properties of the box and the medium to know more.

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