**Additive Properties of Measurable Set for Difference Two Measurable Set**

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**ABSTRAC: *This paper will carry out the problem from [7] to be related difference of two measurable set. The***

***problem is to prove the theorem if A and B are measurable sets such that*** � ⊂ � ***and*** 𝒎 � < ∞ ***then m( A – B) =***

***m(A) – m(B). theorem proving is done through the study of properties measurable set.***

**Keywords---** Aditif, Meaurable, difference two measurable set

**1. INTRODUCTION**

The development of modern measurement theory is characterized by the introduction of the concept outer measure by Henry Lebesgue in 1940. At that time outer measure is defined as the infimum of the total length of the intervals that

cover the set ([7, p 55). With outer measure are introduced, has many problems that can be solved. The examples if the

set is interval then it’s outer measure is equal with interval length, but theoretically outer measure to have a weakness,

because the outer measure do not meet the additive properties that *m \* (A* ∪ *B) ≠ m \* (A) + m \* (B).* That's why the

researchers tried to cover up the weakness of the outer measure. Among researchers it is Henry Lebesgue, which defines

the measure by using the concept outer measure.

By using the concept of measure, important issues that exist in the analysis can be developed such as in ([4], p313)

about the properties of the open set, that the union of an arbitrary collection of open subsets in R is open in R, and on

another litertur is ([5], p136) *if* ��1 *and* ��2 *are open subset of R, then* ��1 ∩ ��2 *also open*. By using the concept of the

measure, ([6], p20) give generalization abaut union of measurable set, that is the union of a sequence of measurable sets

is measurable. Even problems in Real Analysis is not applicable, by using the concept of the measure, the problem can be proven to be valid. The example that *if A and B are open sets in R, then A - B is not necessarily open set in R*, using the concept of measure can be shown that *if A and B are measurable sets then A - B is measurable*.

Discussion outer measure of a set associated with the power set of the set, such as defining the outer measure of ([1]

as follows, v will designate a finite-valued, finitely subadditive outer measure defined on the power set P(X) of a

nonempty set X. 𝜌 will designate the associated set function ��(E) = v(E) – v(E’), where E ⊂ X. defining the outer

measure can also define from the length of open interval as in the following definition. Suppose F is a collection of

countable open intervals. For any J ∈ F, the total

�∈� �(�)

is a positive real number. Let E be any set, take a subset C of

F to C is a collection of J from open intervals �𝑖 such that E⊂

𝑖 �𝑖 . If the set C is written C = {J: J ⊂ F and J cover

E}. Outer measure m \* (E) of the set E is m\*(E) = inf {

𝑖 �(�𝑖 ) : {�𝑖 } open interval and E ⊂

𝑖 �𝑖 .} ([7], p55).

Defining the measure set by [3*] if v is outer measure, then Sv the v-measurable sets = {E* ⊂ *X/ v(G) = v(G*∩*E) +*

*v(G*∩*E’)} for all G* ⊂ *X*. Definition of the measurable set, according to [7] is as follows, the set E is said measurable, *if* ∀

*A* ⊂ *R, apply m\*(A) = m\*(A*∩*E) + m\*(A*∩ �𝑐 *)*, and if E is measurable set, then *m \* (E) = m (E)*. Meanwhile, according to

([2], p115), *if A and B are measurable sets, then m(A*∪ �) *= m(A) + m(B) – m(A*∩ �*)*. This means that *m (A* ∪ *B) ≤ m (A)*

*+ m (B) for an measure set A and B*,

This paper will carry out the problem from ([7], p68) to be related difference of two measurable set. The problem is to

prove the theorem *if A and B are measurable sets such that* � ⊂ � *and* � � < *∞ then m( A – B) = m(A) – m(B)*.

theorem proving is done through the study of properties measurable set.

**2. PROPERTIES OF MEASURABLE SET**

*Theorem* 1. *If E is measurable set then* �𝑐 *is measurable set.*

Proof: Because E is measurable set, by definition ∀ A ⊂ R, we have m\*(A) = m\*(A∩E) + m\*(A∩ �𝑐 ) = m\*(A∩ �𝑐 ) +

m\*(A∩E) = m\*(A∩ �𝑐 ) + m\*(A∩ (�𝑐 )𝑐 )

So �𝑐 is measurable set.

*Theorem* 2 *if D and E measurable set, then D*∩*E measurable*

Proof: Because D is measurable set, by definition, ∀ A ⊂ R,

We have m\*(A) = m\*(A∩D) + m\*(A∩ �𝑐 ).

= m\*((A∩ �) ∩ �) + m\*((A∩D)∩ �𝑐 ) + m\*(A∩ �𝑐 ).

= m\*(A∩(D∩E)) + m\*(A∩ �𝑐 ) + m\*(A∩(D∩ �𝑐 ))

≥ m\*(A∩(D∩E)) + m\*(A∩(�𝑐 ∪ �𝑐 )

Because A∩ (�𝑐 ∪ �𝑐 ) = (A∩ �𝑐 ) ∪ (� ∩ (� ∩ �𝑐 )

= m\*(A∩(D∩E)) + m\*(A∩ (� ∩ �)𝑐 )

then m\*(A) ≥ m\*(A∩(D∩E)) + m\*(A∩ (� ∩ �)𝑐 ) ………………………..1)

Because A = (A∩ (� ∩ �)) ∪ (� ∩ (� ∩ �)𝑐 ), then

m\*(A) ≤ m\*((A∩ (� ∩ �) + m\*(� ∩ (� ∩ �)𝑐 )………………………..2)

from 1) and 2) we have m\*(A) = m\*((A∩ (� ∩ �) + m\*(� ∩ (� ∩ �)𝑐 )

So D∩ � measurable

*Theorem* 3 *If {E1, E2, …, En} are finite collection of measurable set, then* ∪� � *is measurable*

𝑖 =1 𝑖

Proof : We use induction of n, as below;

- Show true for n = 1

1 1

for � = 1 we have ∪𝑖 =1 �1 = E1, because E1 measurable, then ∪𝑖 =1 �1 measurable (true)

- Assume true for n = k-1

for � = � − 1 we have ∪� −1 � measurable

𝑖 =1 𝑖

- It will be proved ∪� � measurable

𝑖 =1 𝑖

� � = � ∪ �

∪ … ∪ � ∪ �

∪𝑖 =1 𝑖 1 2

� −1 �

� −1 � ∪ �

= ∪��=1 𝑖 �

� −1

Because ∪𝑖 =1 �𝑖 measurable, and Ek measurable, then

� −1 � ∪ �

=∪� �

measurable.

∪𝑖 =1 𝑖 �

𝑖 =1 𝑖

�

*Theorem* 4 *If {E1, E2, …, En} are finite collection of measurable set, then* ∩𝑖 =1 �𝑖 *is measurable*

Proof: We use induction of n, as below;

- Show true for n = 1

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for � = 1 we have ∩𝑖 =1 �1 = E1 because E1 measurable, then ∩𝑖 =1 �1 measurable (true)

- Assume true for n = k-1

for � = � − 1 we have ∩� −1 � measurable

𝑖 =1 𝑖

- It will be proved ∩� � measurable

𝑖 =1 𝑖

� � = � ∩ �

∩ … ∩ � ∩ �

∩𝑖 =1 𝑖 1 2

� −1 �

� −1 � ∩ �

= ∩��=1 𝑖 �

� −1

� −1 �

Because ∩𝑖 =1 �𝑖 measurable, and Ek measurable, then ∩𝑖 =1 �𝑖 ∩ �� =∩𝑖 =1 �𝑖 measurable.

*Teorema* 5 *Let* �1 , �2 , �3 , … , �� *are finite sequance from disjoint measurable sets, then for arbitrary set A.*

�

�∗ � �𝑖

�=1

�

= �∗ � ∩ �𝑖

𝑖 =1

Proof: We use induction,

1) for � = 1 thru, because �∗ �

1

�=1

�𝑖

= �∗ � ∩ �1 =

1

𝑖 =1

�∗ � ∩ �𝑖

2) Assume the statement is true for � = � − 1 with 1 < � ≤ �, then

�∗ �

�=1 =

� −1 �𝑖

� −1 ∗

𝑖 =1

�

� ∩ �𝑖

(1)

3) It will be proved that the equation is also true for � = � with 1 < � ≤ �.

Consider the equation (1). With added �∗ (� ∩ �� ) for equation (1) at two side,

� −1

�∗ � �𝑖

�=1

� −1

+ �∗ (� ∩ �� ) = �∗ � ∩ �𝑖 + �∗ (� ∩ �� )

𝑖 =1

�

=

𝑖 =1

�∗ � ∩ �𝑖

(2)

Because �1 , �2 , �3 , … , �� disjoint, then

� −1 �𝑖 with �

disjoint. Then

 � −1 �𝑖

𝑖 =1 �

𝑖 =1 ∩ �� = ∅ (3)



Consider

� −1 �𝑖

𝑐 =

𝑖 =1 �

(4)

 � �𝑖 ∩ � =

� −1 � ∪ �

∩ � =

� −1 � ∩ �

∪ � ∩ �

(5)

𝑖 =1

� 𝑖 =1 𝑖 � �

𝑖 =1 𝑖 � � �

 � �𝑖 ∩ �𝑐 =

� −1 � ∪ �

∩ �𝑐 =

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∩ �𝑐 =

� −1 � ∩ �𝑐 (6)

𝑖 =1

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𝑖 =1 𝑖 �

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form (3) with (5) and (4) with (6) we have

� �𝑖

� 𝑐

� −1

𝑖 =1 ∩ �� = �� dan

𝑖 =1 �𝑖 ∩ �� =

𝑖 =1 �𝑖

Then, equation (2) can write

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�=1

�𝑖

∩ �𝑐 + �∗ (� ∩

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�=1

�𝑖

∩ �� ) =

�

𝑖 =1

�∗ � ∩ �𝑖

(7)

Because �1 , �2 , �3 , … , �� measurables, then �� measurable. From definition, and take, set test � ∩

�

�=1

�𝑖 , then

�∗ �

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�=1

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�𝑖

∩ �𝑐 + �∗ � ∩

�

�=1

�𝑖

∩ ��

= �∗ � ∩

�

�=1

�𝑖

(8)

from (7) and (8) then

�

�∗ � ∩ �𝑖

�=1

�

= �∗ � ∩ �𝑖

𝑖 =1

So, base from induction principle, if �1 , �2 , �3 , … , �� are finite sequance from disjoint measurable sets, then for arbitrary

set A, apply

n

m∗ A Ei

I=1

n

= m∗ A ∩ Ei

i=1

*Theorem* 6 *If* �1 , �2 , �3 , … , �� *are finite sequence of disjoint measurable set, then* �*(*

�

𝑖 =1

�𝑖 *) =*

�

𝑖 =1

� (���)

Proof: Because {�1 , �2 , �3 , … , �� } measurable and disjoint, then from theorem 3

�

𝑖 =1

E𝑖 measurable. Take = ℝ , then

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𝑖 =1

E𝑖 ) = �∗ (

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𝑖 =1

E𝑖 ) (outher measure equal with measure)

= �∗ (ℝ  (

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E𝑖

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𝑖 =1

E𝑖 )

= � �∗ (ℝ E𝑖 )

( E , E , … , E

finite sequence of disjoint

𝑖 =1

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measurable set)

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𝑖 =1

�∗ (E𝑖 )

(Because ℝ E𝑖

= E𝑖 )

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𝑖 =1

� (E𝑖 )

(outher measure equal with measure)

So, If �1 , �2 , �3 , … , �� are finite sequence of disjoint measurable set, then �(

�

𝑖 =1

E𝑖 ) =

�

𝑖 =1

� (E��)

**3. PROOF THE PROBLEM**

*Theorem* 7 *If E1 and E2 are measurable sets, such that* �2 ⊂ *E1 and m(E2) <* ∞*, then m(E1 – E2) = m(E1) – m(E2).*

Proof: Consider that: E1 – E2 =E1 ∩ E2c

Let: E1 = E1 ∩ R

= E1 ∩ ( E2c ∪E2 )

= ( E1 ∩ E2c ) ∪ ( E1 ∩ E2) Because E2 ⊂ E1

= ( E1 ∩ E2c )∪ E2

Because E1 and E2 are measurable sets, base from theorem , then E c

2

c

is measurable.

Base theorem 2 because E1 and E2c are measurable, then E1 ∩ E2

c

is measurable.

E1= ( E1 ∩ E2

Claim: ( E1 ∩ E2c ) ∩ E2 = Ø

) ∪ E2

proof claim: ( E1 ∩ E2c ) ∩ E2 = ( E1 ∩ E2 ) ∩ ( E2c ∩ E2 ) = E2 ∩ Ø = Ø

Because ( E1 ∩ E2c ) ∩ E2 = Ø dan E1= ( E1 ∩ E2c ) ∪ E2

Base theorem 6 we have

m ( Ei ) = m (( E1 ∩ E2c ) ∪ ( E2 ))

m ( E1 ) = m ( E1 ∩ E2c ) + m ( E2 )

m ( E1 ) = m ( E1 - E2 ) + m ( E2 ) karena E1 – E2 = E1 ∩ E2c

m ( E1 ) - m ( E2 ) = m ( E1 - E2 ).

So, m ( E1 - E2 ) = m (E1 ) - m (E2 ).

**4. CONCLUTION**

A new properties of measurable set have been discovered recently. Properties of measurable set have attracted researchers of the field to investigate these newly discovered properties in detail. This article investigate the properties of

two measurable set that m(E1 – E2) = m(E1) – m(E2).

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